MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2016/2017

DIM5058 –MATHEMATICAL TECHNIQUES 1

(RS)

26 MAY 2017

9.00 a.m. – 11.00 a.m.

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages with **4** questions. Key formulae are given in the Appendix.

2. Answer **ALL** questions.

3. Write your answers in the answer booklet provided.

4. All necessary working steps must be shown.

Question 1

a) Solve x for $\frac{x+2}{3-x}=\frac{5}{4}$. (3 marks)

b) Solve the following quadratic functions below using the specified method.

i) $x^2=12-x$, by using **Quadratic Formula.** (5 marks)

ii) $4x^2-16x-84=0$, by using **Completing the Square Method.** (6 marks)

c) Solve the following inequalities, and express your answer in an **interval notation.**

i) $1+6x \le x+16$ (5 marks)

ii) $|2x+3|>15$ (6 marks)

[TOTAL 25 MARKS]

Question 2

a) Given that $f(x)=\frac{5}{x+1}$ and $g(x)=7x-2$.

i) Find $(g \circ f)(x)$ and express in a simplified form. (3 marks)

ii) Find the domain of $(g \circ f)(x)$. (2 marks)

iii) Evaluate $(g \circ f)(3)$. (2 marks)

b) Given a quadratic function $f(x)=x^2+4x-21$.

i) Find the x-intercept(s). (4 marks)

ii) Find the y-intercept. (2 marks)

c) Given a rational function $f(x)=\frac{5x-1}{x+2}$.

i) Find the horizontal and vertical asymptotes. (1 mark)

ii) Find the coordinate of x-intercept. (2 marks)

iii) Find the coordinate of y-intercept. (2 marks)

iv) State the coordinate when x = - 4 and 6. (4 marks)

iv) Sketch the graph and label all your answers that you get from (i) – (iv). (3 marks)

[TOTAL 25 MARKS]

Continued...

Question 3

a) Given matrix $P=\begin{bmatrix}3 & 0 & -1\\ 5 & 8 & 16\end{bmatrix}$, $Q=\begin{bmatrix}-3 & 9 & 0\\ 0 & 12 & 9\end{bmatrix}$ and $R=\begin{bmatrix}2 & 1\\ 1 & 1\\ 4 & 3\end{bmatrix}$. Compute

i) $(P-Q)^T$ (3 marks)

ii) QR (4 marks)

b) Mr. Jacob produces two types of mangoes known as Chokanan and Harumanis. The total profits that he gets by selling the mangoes in April is RM612, while in May is RM940. The following table shows the weight (in *kg*) of the mangoes sold for both months.

	Chokanan	**Harumanis**	**Total profit**
April	22	15	612
May	18	26	940

[Hint: Let x and y be the price for Chokanan and Harumanis respectively.]

i) Represent the above information in the matrix form of $\boldsymbol{AX}=\boldsymbol{B}$. (2 marks)

ii) Find A^{-1}. (3 marks)

iii) From part (i) and (ii), solve the value of x and y by using the inverse matrix method. (4 marks)

c) For the following system of linear equation:

$$\begin{cases} 6x+3y-9z=66 \\ 2x-2y+18z=-15 \\ 5y-3z=83 \end{cases}$$

i) Transform the equations into matrix form of $\boldsymbol{AX}=\boldsymbol{B}$. (2 marks)

ii) Use *Cramer's Rule* method to solve for z. (7 marks)

[TOTAL 25 MARKS]

Continued...

Question 4

a) Find the sum of the sequence $\sum_{n=1}^{4} \frac{(n+2)^2}{n!}$. (5 marks)

b) Given a sequence of $-5,2,9,16,....$ Find the sum of the first twenty terms. (5 marks)

c) Sara borrows money from Amy. She promises to pay back the money on a monthly basis for 6 months. The first instalment is RM15. Every instalment of a month is twice that of the previous month.

i) State the common ratio. (1 mark)

ii) How much is the final instalment? (2 marks)

iii) How much is the amount of money that Sara borrowed? (2 marks)

d) Given a series of $6+\frac{6}{5}+\frac{6}{25}+\frac{6}{125}+....$

i) Find the common ratio. (1 mark)

ii) Does this infinite series have a sum? If yes, find the value of S_∞. (3 marks)

e) Expand $\left(x^2+2x\right)^4$. (6 marks)

[TOTAL 25 MARKS]

End of Page.

Appendix

Inequalities:

$\|u\| < a \quad \Rightarrow \quad -a < u < a$	$\|u\| > a \quad \Rightarrow \quad u < -a \quad or \quad u > a$
$\|u\| \leq a \quad \Rightarrow \quad -a \leq u \leq a$	$\|u\| \geq a \quad \Rightarrow \quad u \leq -a \quad or \quad u \geq a$

Completing the square:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Quadratic formula:

If $ax^2 + bx + c = 0$ where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Standard form of a quadratic function:

$f(x) = a(x - h)^2 + k,\ a \neq 0$

Trigonometric functions:

$y = A \sin(Bx - C)$ or $y = A \cos(Bx - C)$

amplitude $= |A|$, period $= \dfrac{2\pi}{B}$, and phase shift $= \dfrac{C}{B}$.

Determinant of a 2 × 2 matrix	*Determinant of a 3 × 3 matrix*
$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$
Inverse of a 2 × 2 matrix	***Inverse of a 3 × 3 matrix***
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $ad - bc \neq 0$.	$A^{-1} = \dfrac{1}{\|A\|}[c_{ij}]^T$ $A^{-1} = \dfrac{1}{\|A\|}\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$ $A^{-1} = \dfrac{1}{\|A\|} adj\, A$ where $[c_{ij}]^T$ is called the adjoint of A (adj A). c_{ij} of the entry $a_{ij} = (-1)^{i+j} M_{ij}$

Cramer's Rule for 2 × 2 matrix	*Cramer's Rule for 3 × 3 matrix*
If $\begin{aligned} a_1x+b_1y&=c_1\\ a_2x+b_2y&=c_2 \end{aligned}$ then $x=\dfrac{\begin{vmatrix} c_1 & b_1\\ c_2 & b_2\end{vmatrix}}{\begin{vmatrix} a_1 & b_1\\ a_2 & b_2\end{vmatrix}}$ and $y=\dfrac{\begin{vmatrix} a_1 & c_1\\ a_2 & c_2\end{vmatrix}}{\begin{vmatrix} a_1 & b_1\\ a_2 & b_2\end{vmatrix}}$ where $\begin{vmatrix} a_1 & b_1\\ a_2 & b_2\end{vmatrix}\neq 0$	If $\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3 \end{aligned}$ then $x=\dfrac{D_x}{D}, y=\dfrac{D_y}{D}, z=\dfrac{D_z}{D}$ where $D=\begin{vmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$, $D_x=\begin{vmatrix} d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix}$, $D_y=\begin{vmatrix} a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix}$, $D_z=\begin{vmatrix} a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3\end{vmatrix}$

Arithmetic sequence	*Geometric sequence*
$a_n=a_1+(n-1)d$ $S_n=\dfrac{n}{2}(a_1+a_n)$	$a_n=a_1r^{n-1}$, $S_n=\dfrac{a_1(1-r^n)}{1-r}$ $S_\infty=\dfrac{a_1}{1-r}, \lvert r\rvert<1$

Binomial Theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k;\quad n\geq 1$$

The $(r+1)^{st}$ term in the expansion of $(a+b)^n$ is $\binom{n}{r}a^{n-r}b^r$.